



Qualitative Studies with Microwaves

Physics 401, Fall 2016

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Qualitative Studies with Microwaves

The main goals of the Lab:

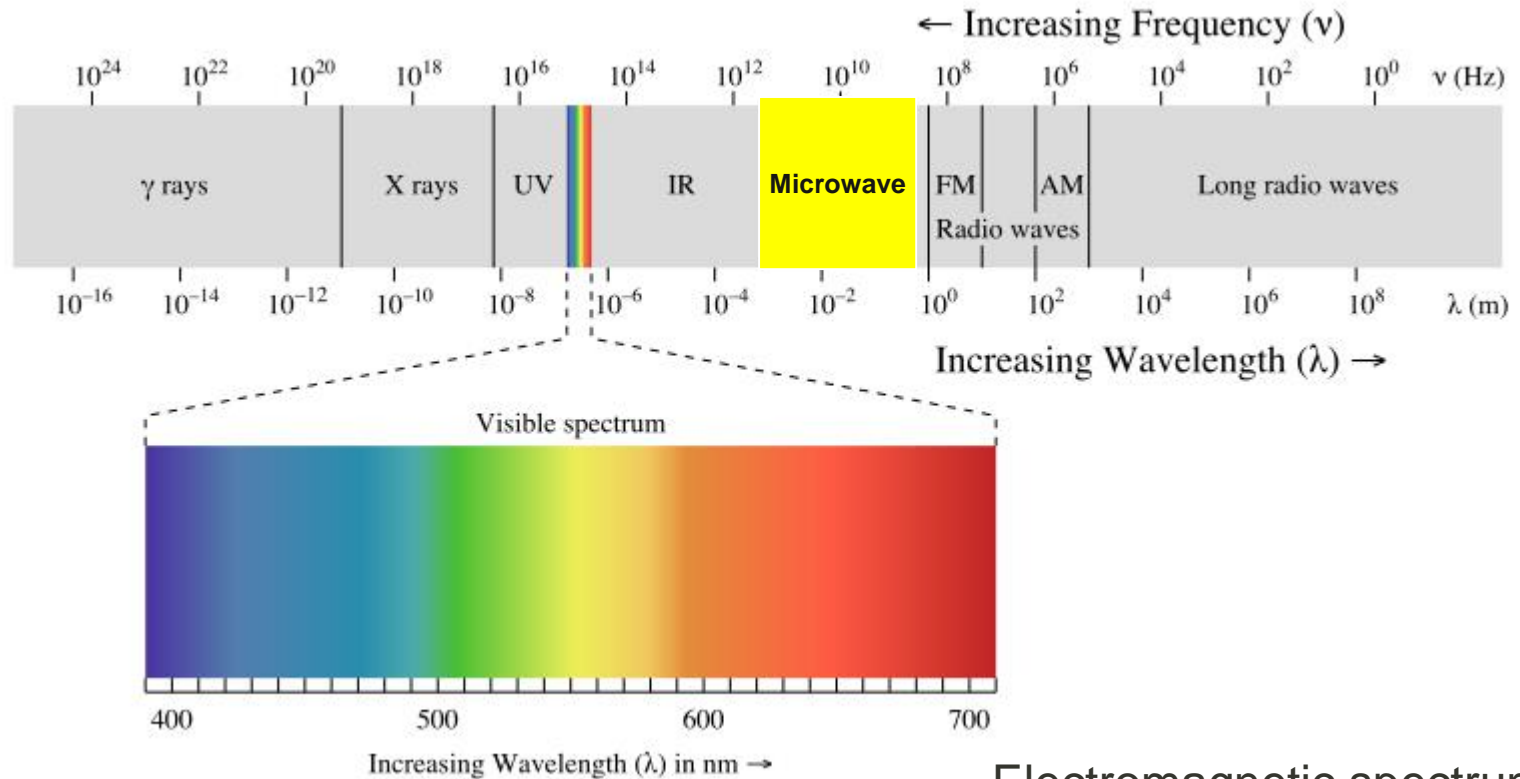
- ✓ Refreshing the memory about the electromagnetic waves propagation
- ✓ Microwaves. Generating and detecting of the microwaves
- ✓ Microwaves optic experiments

This is two weeks Lab



Microwaves place in the electromagnetic spectrum

The microwave range includes ultra-high frequency (**UHF**) (0.3–3 GHz), super high frequency (**SHF**) (3–30 GHz), and extremely high frequency (**EHF**) (30–300 GHz) signals.



Application of the microwaves



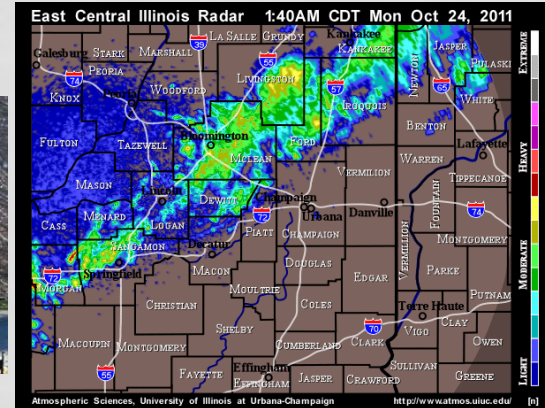
Microwave oven (2.45GHz)



Communication (0.8-2.69GHz)



Satellite TV (4-18GHz)



Weather radar (8-12GHz)



Radar (up to 110GHz)



Motion detector (10.4GHz)



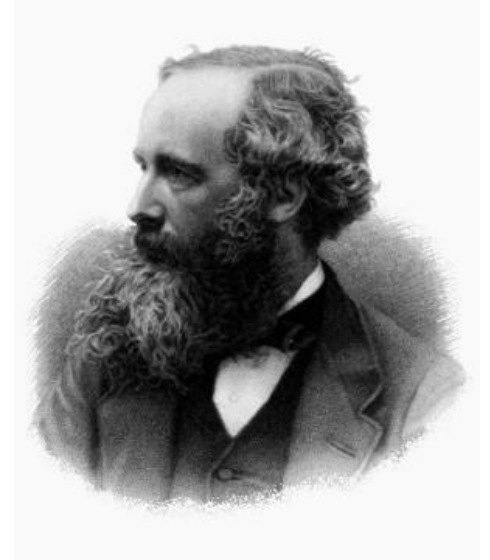
Maxwell equations

$$\nabla \vec{D} = \rho \quad (1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \vec{B} = 0 \quad (2)$$

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad (4)$$



James Clerk Maxwell
(1831–1879)

If $\rho = 0$ and $J = 0$ and taking in account that $\vec{D} = \epsilon \vec{E}$
 $\vec{B} = \mu \vec{H}$ (1) and (4) can be rewritten as

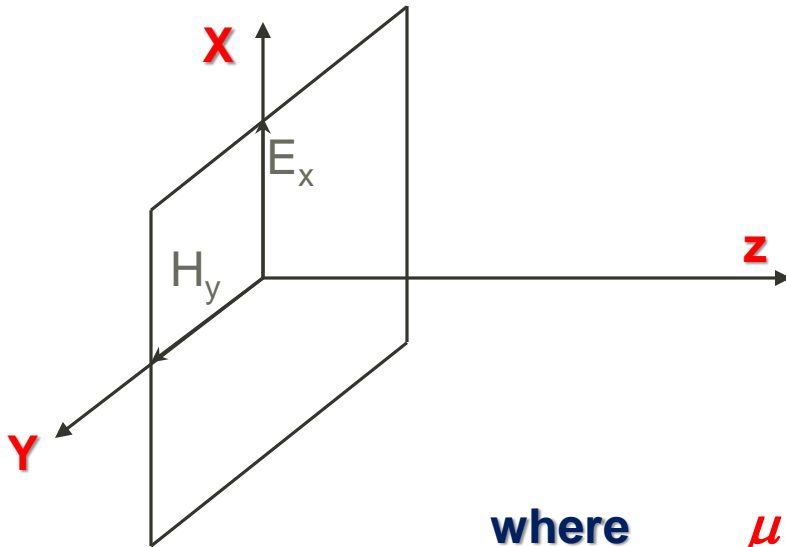
$$\nabla \vec{D} = \epsilon \left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = 0 \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$



Plane wave

Now assuming that plane wave propagate in z direction and what leads to $E_y=E_z=0$ and $H_x=H_z=0$

Now (3) and (4) could be simplified as



$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad (5)$$

$$\frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t} \quad (6)$$

where $\mu = \mu_0 \mu_r$ $\epsilon = \epsilon_0 \epsilon_r$

μ_0 is the free space permeability, ϵ_0 is the free space permittivity
 μ_r is permeability of a specific medium, ϵ_r is permittivity of a specific medium

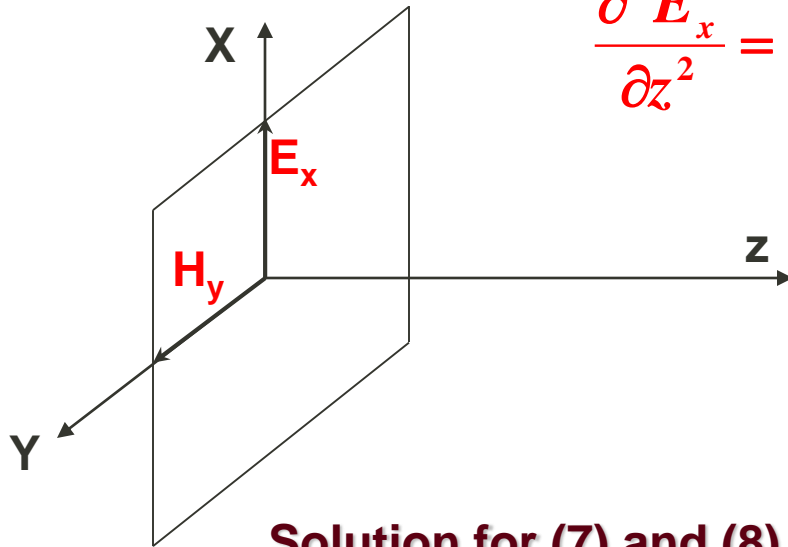


Plane wave

Combining (5) and (6) (see Lab write-up for more details) we finally can get the equations of propagation of the plane wave:

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \quad (7) \quad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 H_y}{\partial t^2} \quad (8)$$

where $v = \frac{1}{\sqrt{\epsilon\mu}}$



$$E_x = E_{x0} \cos(\omega t - kx)$$

$$H_y = H_{y0} \cos(\omega t - kx)$$

Solution for (7) and (8) can found as $H_y = \sqrt{\frac{\epsilon}{\mu}} E_x$ or $E_x = Z H_y$

where $Z = \sqrt{\frac{\mu}{\epsilon}}$ known as characteristic impedance of medium

k is wave vector and is defined as $k = \frac{2\pi}{\lambda}$ or $k = \frac{\omega}{v}$

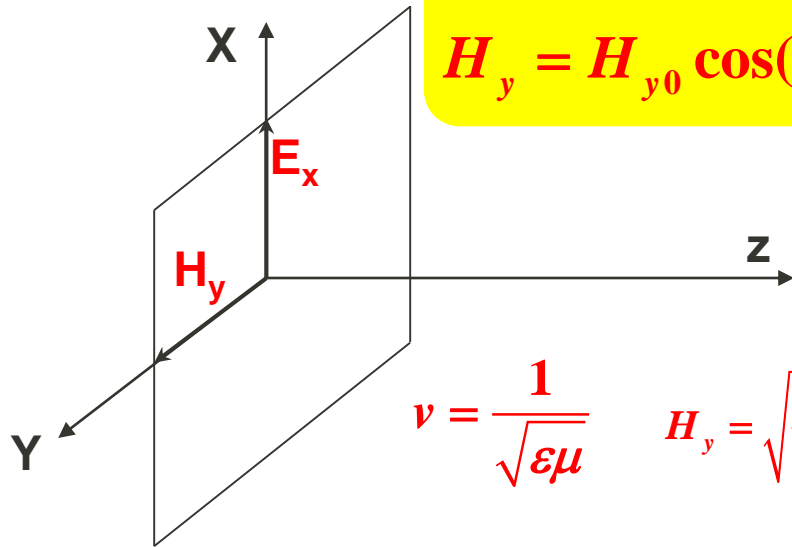
For free space ($\epsilon_r=1$ and $\mu_r=1$) $Z_{fs} = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \text{ohms}$



Plane wave

$$E_x = E_{x0} \cos(\omega t - kx)$$

$$H_y = H_{y0} \cos(\omega t - kx)$$

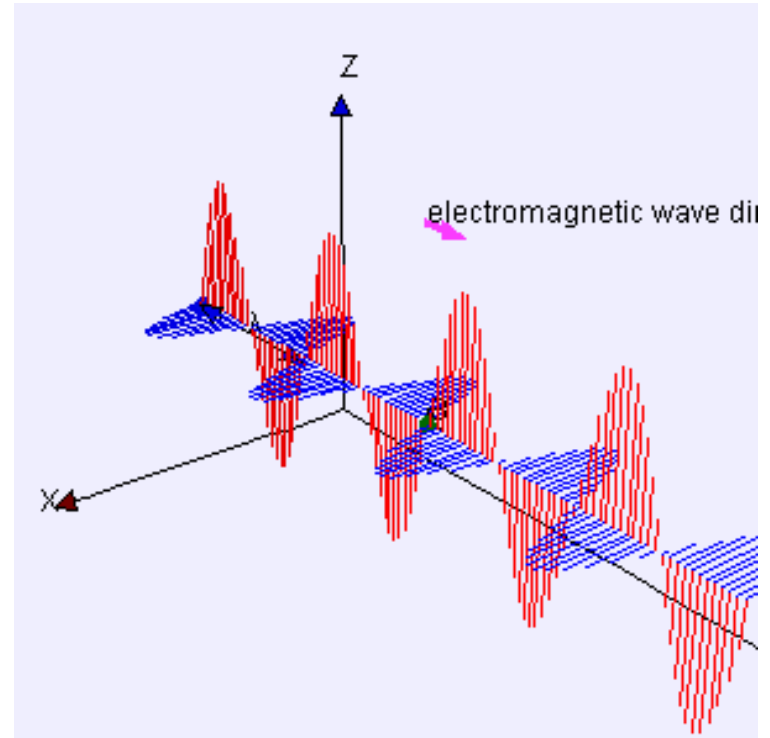


$$v = \frac{1}{\sqrt{\epsilon\mu}} \quad H_y = \sqrt{\frac{\epsilon}{\mu}} E_x$$

$$Z = \sqrt{\frac{\mu}{\epsilon}} \quad E_x = ZH_y \quad k = \frac{2\pi}{\lambda} \quad \text{or} \quad k = \frac{\omega}{v}$$

$$Z_{fs} = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377 \text{ ohms}$$

For free space ($\epsilon_r=1$ and $\mu_r=1$)



*by courtesy Wikipedia



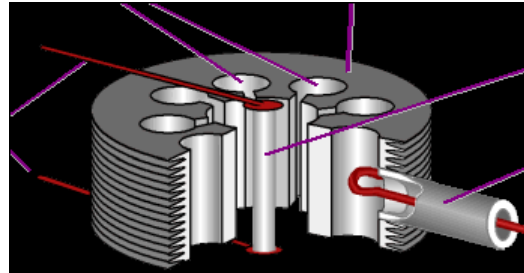
Generating of the microwaves

Vacuum tubes: klystron, magnetron, traveling wave tube

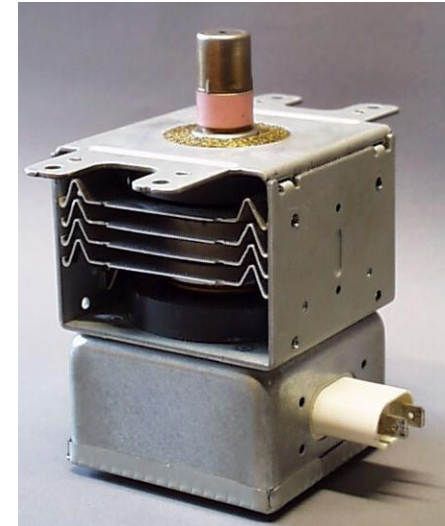
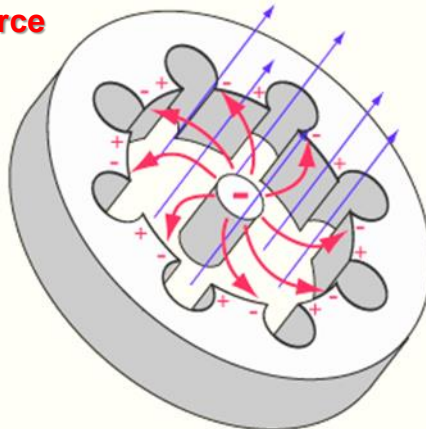
Solid state devices: FET, tunneling diodes, Gunn diodes



Tunable frequency from 9 to 10GHz; maximum output power 20mW



Heated cathode as electron source



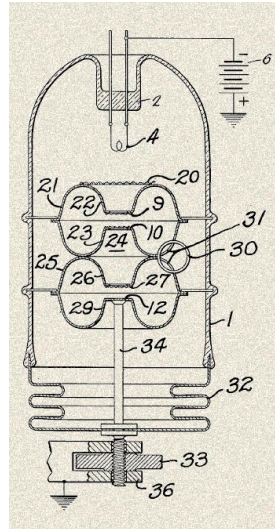
Microwave oven magnetron; typical power 0.7-1.5kW

Klystron. A piece of history.



**Russell Harrison
Varian** (April 24, 1898
– July 28, 1959)

**Sigurd Fergus
Varian** (May 4, 1901
– October 18, 1961)



Patented May 20, 1941

2,242,275

UNITED STATES PATENT OFFICE

2,242,275

ELECTRICAL TRANSLATING SYSTEM AND METHOD

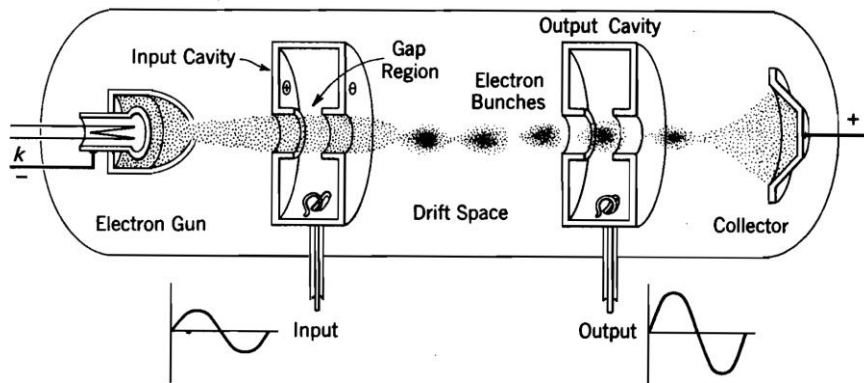
Russell H. Varian, Stanford University, Calif., as-
signor to The Board of Trustees of The Leland
Stanford Junior University, Stanford Uni-
versity, Calif., a corporation of California

Application October 11, 1937 Serial No. 162,955

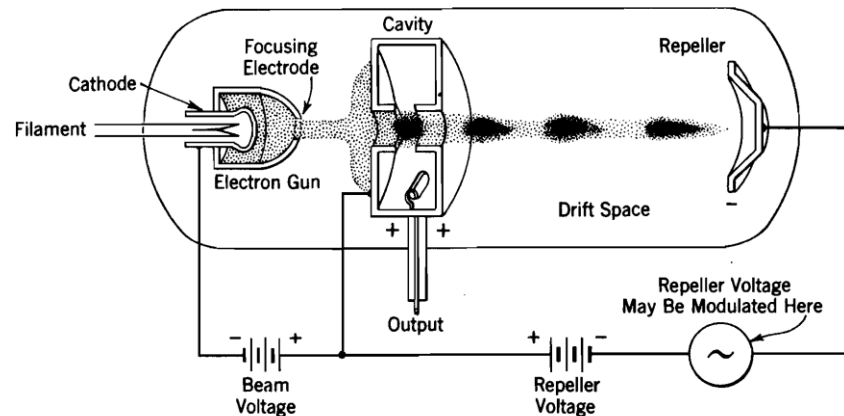
Varian Brothers...Klystron Tube (1940)



Generating of the microwaves. Klystron.



Single transit klystron



Reflection klystron

Advantages: well defined frequencies, high power output

High power klystron used in Canberra Deep Space Communications Complex (courtesy of Wikipedia)



2K25 Klystron



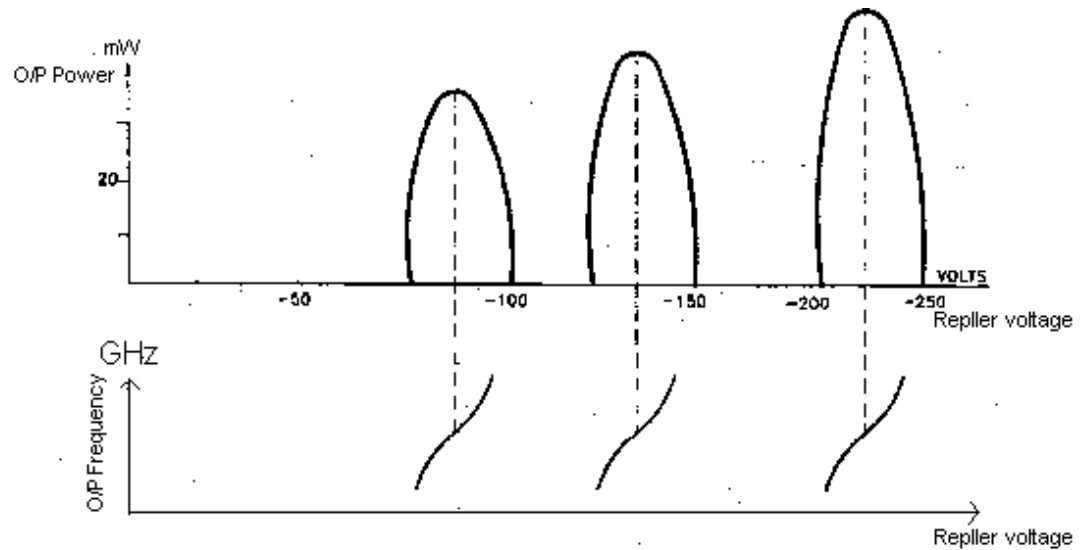
GENERAL CHARACTERISTICS

Frequency Range8,500 to 9,660 Mc

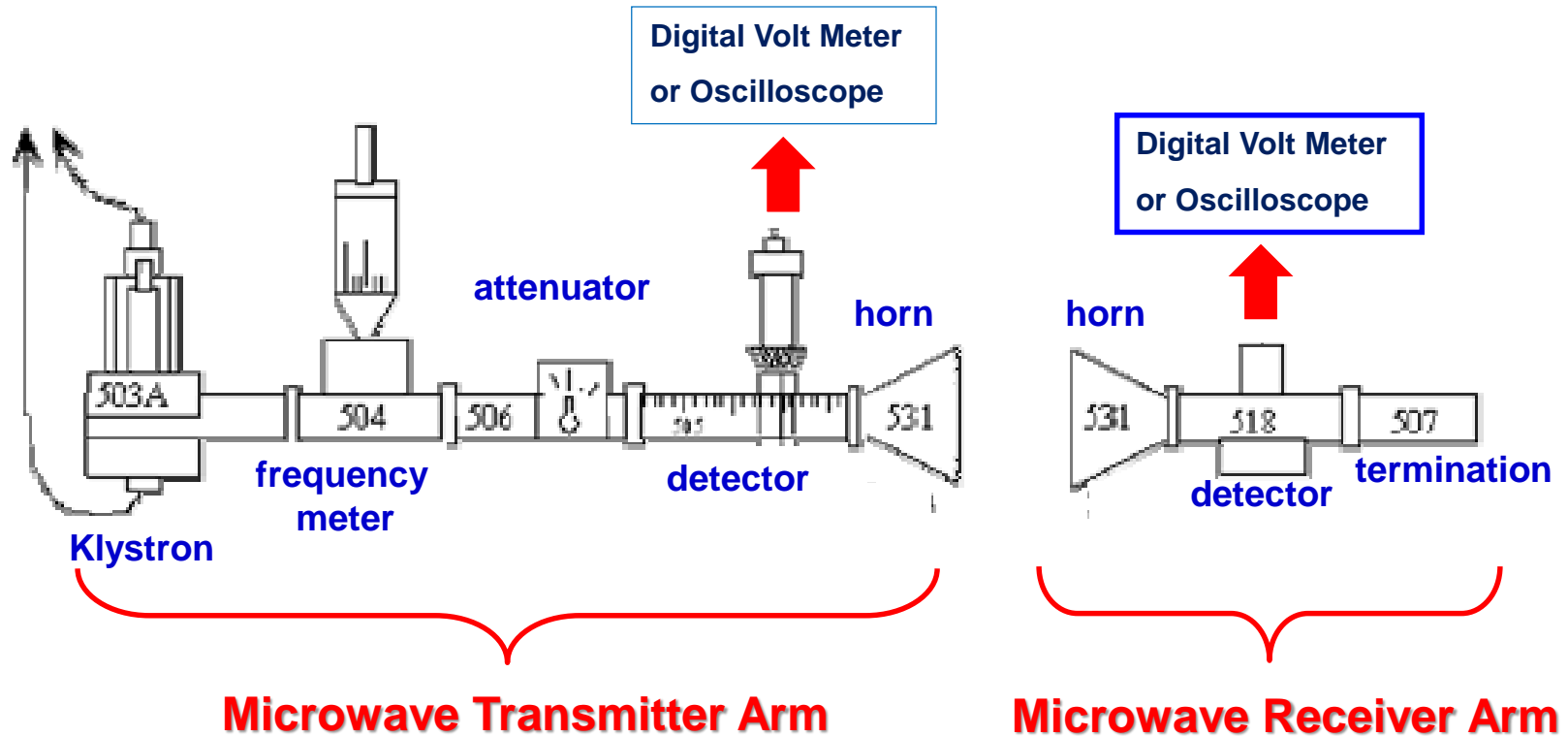
Cathode Oxide-coated, indirectly heated

Heater Voltage.....6.3Volts

Heater Current.....0.44 Amperes



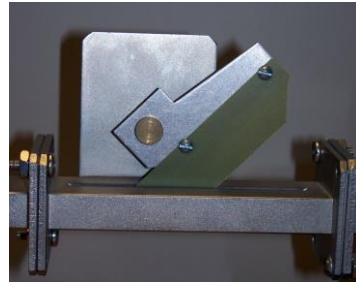
Experimental setup. Main components.



Experimental setup. Main components.



Klystron



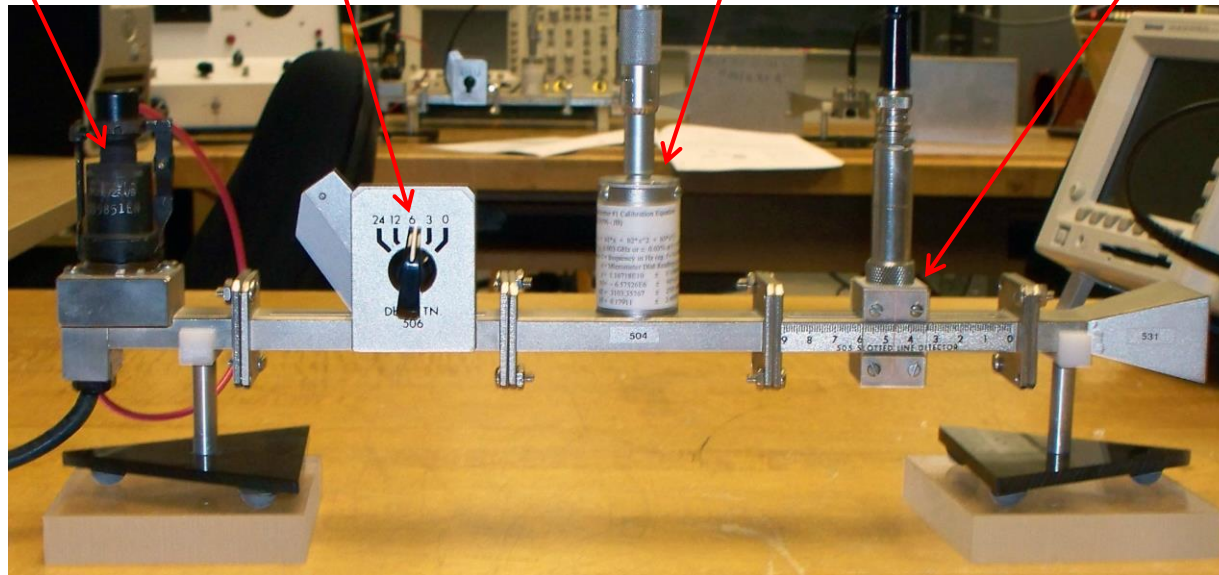
Attenuator



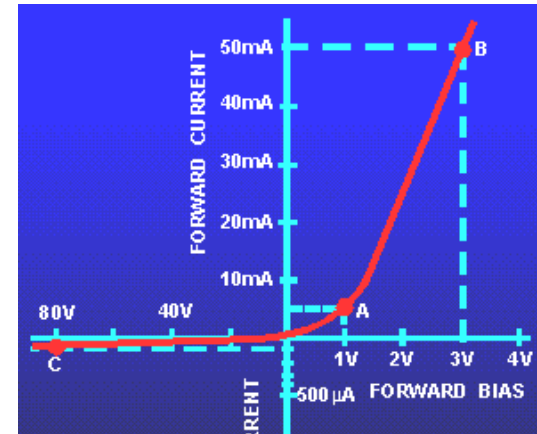
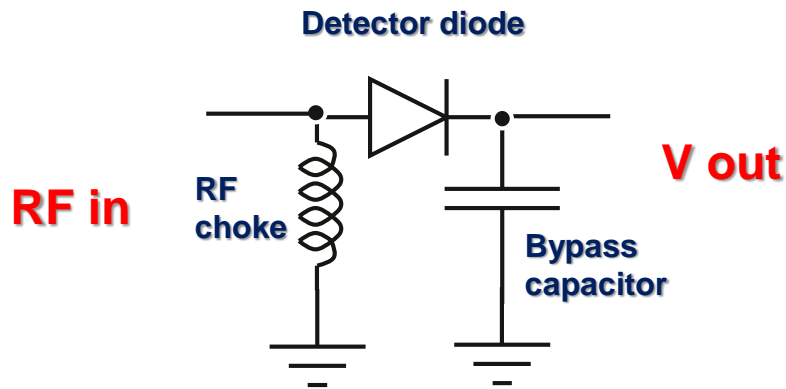
Frequency meter



detector



Detecting of the microwaves



Typical I-V dependence for p-n diode

$$I = I_0 \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

Taylor expansion for exp function will give

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$I \propto aV + bV^2 + \dots$$

0(DC)

If $V = V_0 \sin \omega t$

$$b * \frac{V_0^2}{2} (1 - \cos 2\omega t)$$

$$I_{DC} \propto b \frac{V_0^2}{2} + \dots$$

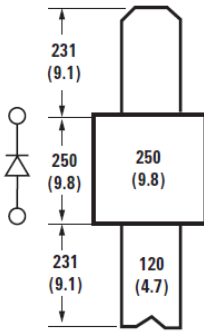
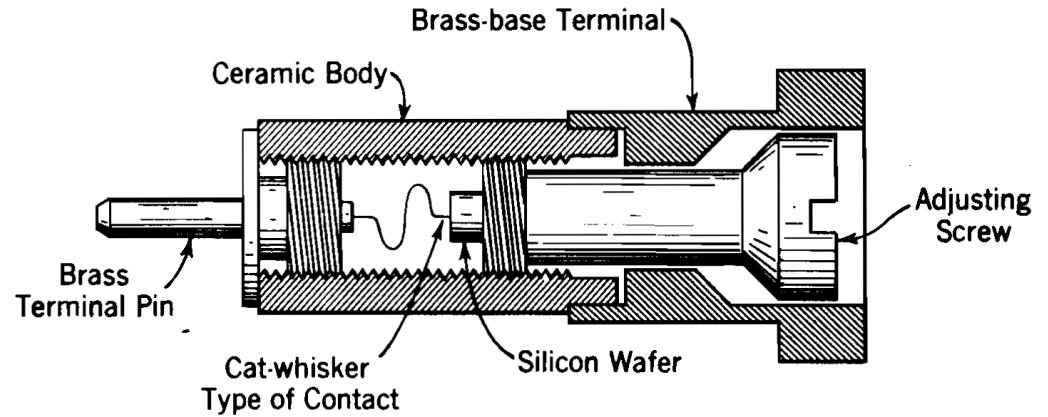
And finally



Detecting of the microwaves

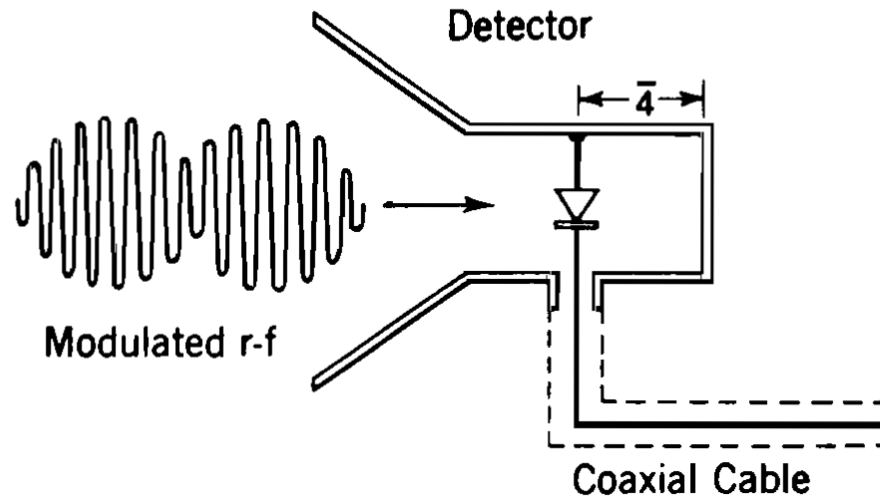
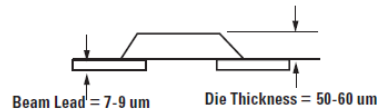


HSCH-9161
HSCH-9162
GaAs Detector Diode

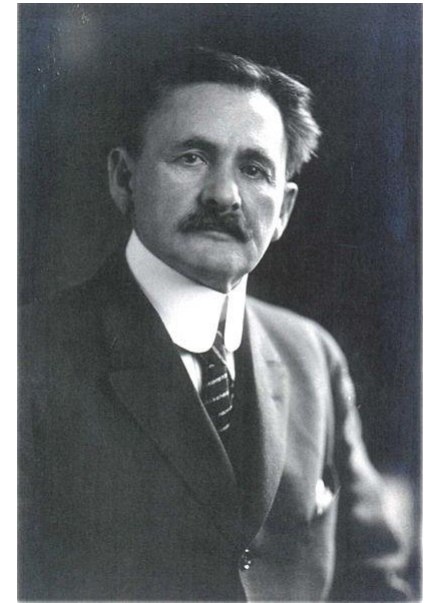


$f_c \sim 200\text{GHz}$

Note: All dimensions in microns (mils)

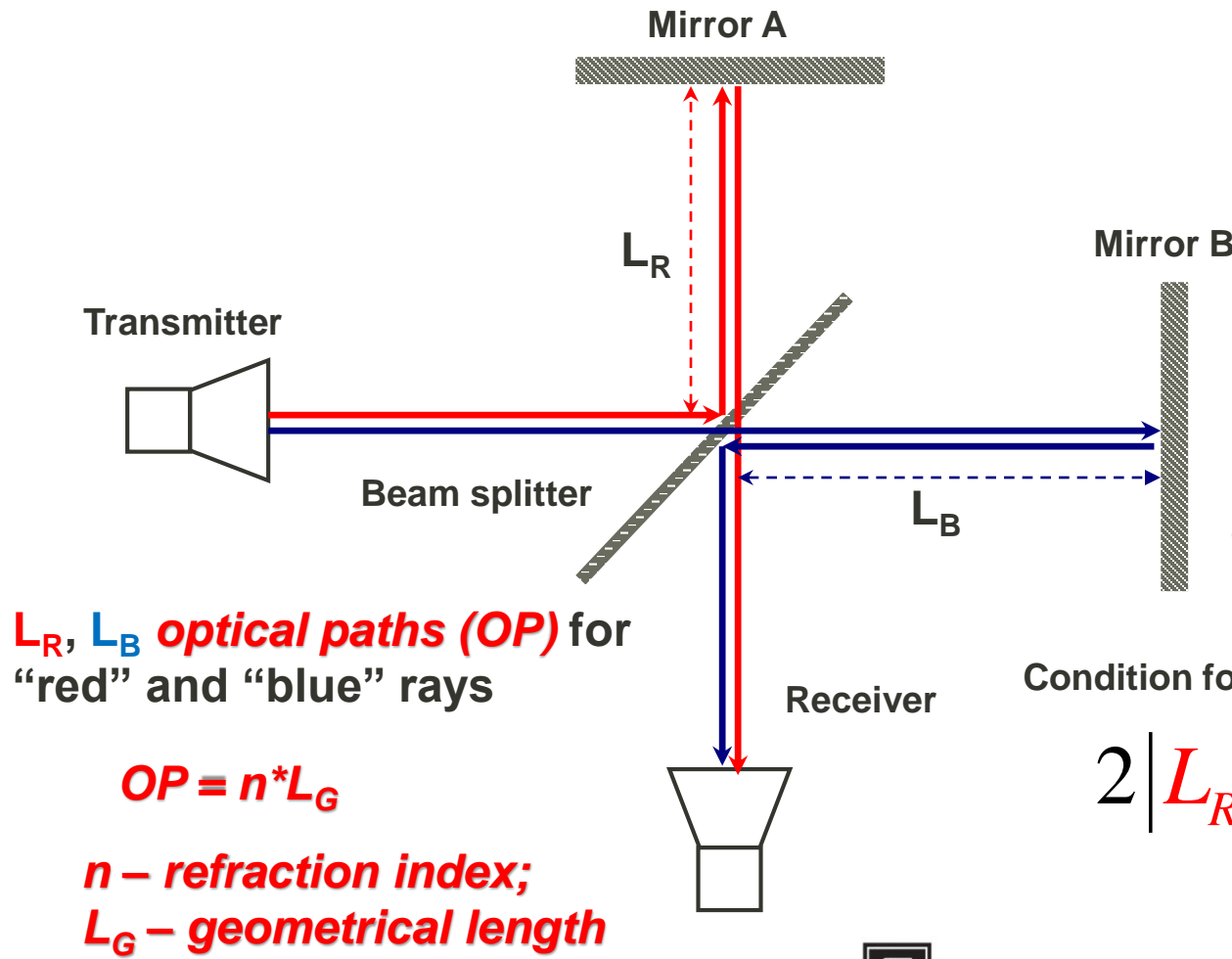


Experiments: Michelson interferometer



Albert Abraham Michelson
(1852 - 1931)

The Nobel Prize in Physics 1907



Condition for constructive interference

$$2 |L_R - L_B| = k\lambda$$



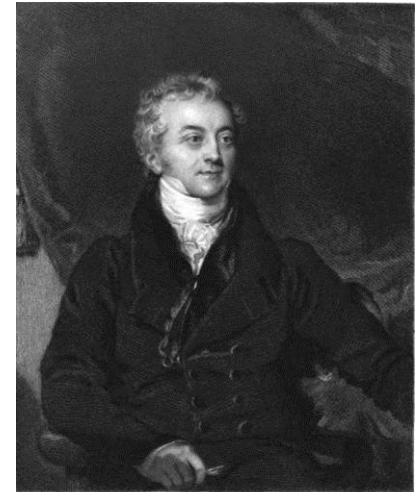
Experiments: Michelson interferometer



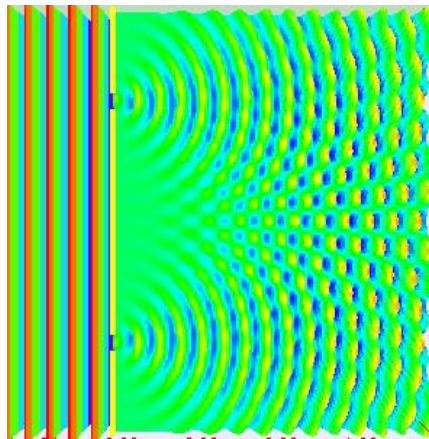
Physics 403 Lab Michelson interferometer setup



Experiments: Double slit Interference. T. Young 1801



Thomas Young
(1773 – 1829)



For constructive
Interference
 $\Delta r = n\lambda$ or $d \sin \theta = n\lambda$

The measured envelope of the diffraction pattern can be defined as:

$$|\psi_{ss}|^2 = |\psi_0|^2 \left(\frac{\sin x}{x} \right)^2 \times \cos^2 [kd \sin(\theta/2)]$$

where $x = kb \sin(\theta/2)$ and

$$k = \frac{2\pi}{\lambda} \text{ is wave vector of the plane wave}$$

$$\Delta r = r_1 - r_2 = d \sin \theta$$

Δr

r_1

r_2

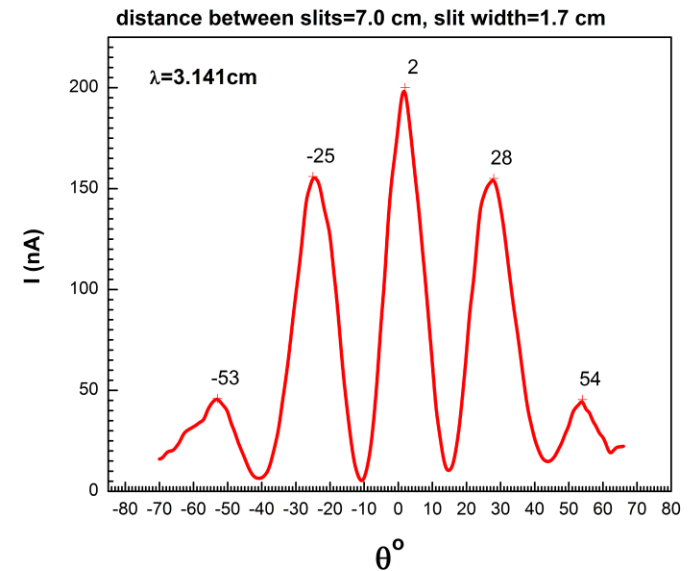
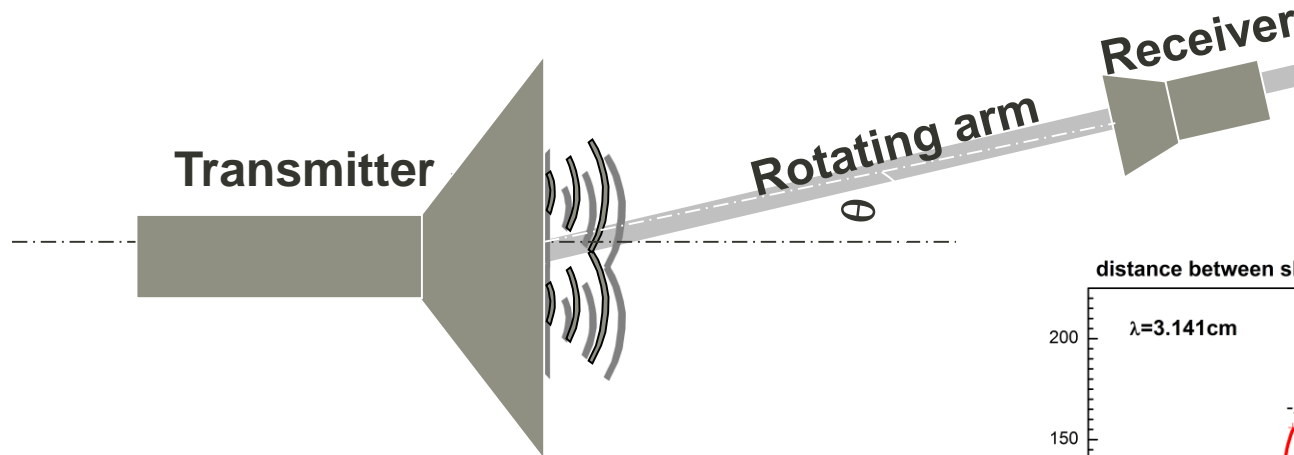
θ

d

b



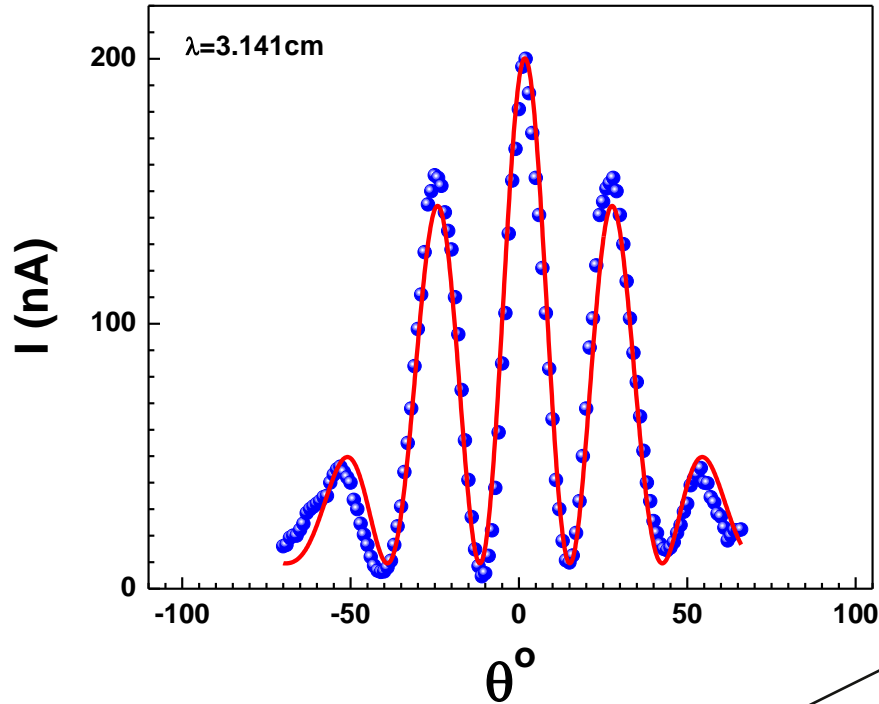
Experiments: Double slit interference



Physics 401 Lab setup and example of the data

Experiments: Double slit interference. Fitting

$$|\psi_{ss}|^2 = |\psi_0|^2 \left(\frac{\sin x}{x} \right)^2 \times \cos^2 [(kd \sin(\theta/2))] \quad x = kb \sin(\theta/2)$$



Model	Two_slit (User)		
Equation	$y=I_0 \cdot \left(\frac{\sin(K1 \cdot \sin(\pi \cdot x/360 + f))}{K1 \cdot \sin(\pi \cdot x/360 + f)} \right)^2 \cdot \cos^2(K2 \cdot \sin(\pi \cdot x/360 + f)) + I_{00}$		
Reduced Chi-Sqr	94.62111		
Adj. R-Square	0.96659	Value	Standard Error
	I0	190.6014	3.042882
	K1	4.384042	0.074754
	K2	13.51332	0.052244
	f	-0.01525	7.19E-04
	I00	9.572049	1.440409

Fitting equation

$$y = I_0 \cdot \left(\frac{\sin(K1 \sin(\frac{\pi x}{360} + f))}{K1 \sin(\frac{\pi x}{360} + f)} \right)^2 \cos^2 \left(K2 \sin \left(\frac{\pi x}{360} + f \right) \right) + I_{00}$$

Here in fitting expression:

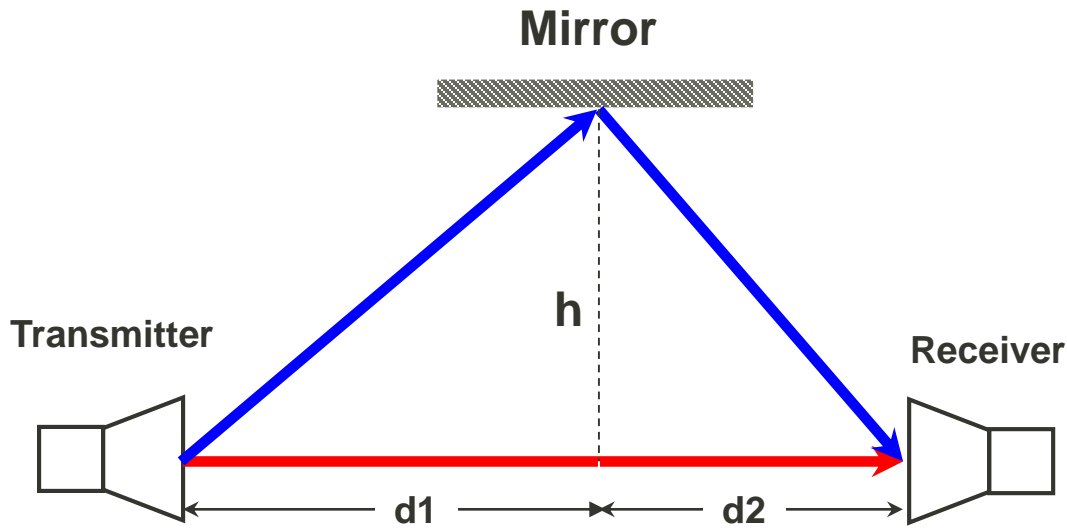
$$I_0 = |\psi_0|^2;$$

$$K1 = kb;$$

$$K2 = kd$$



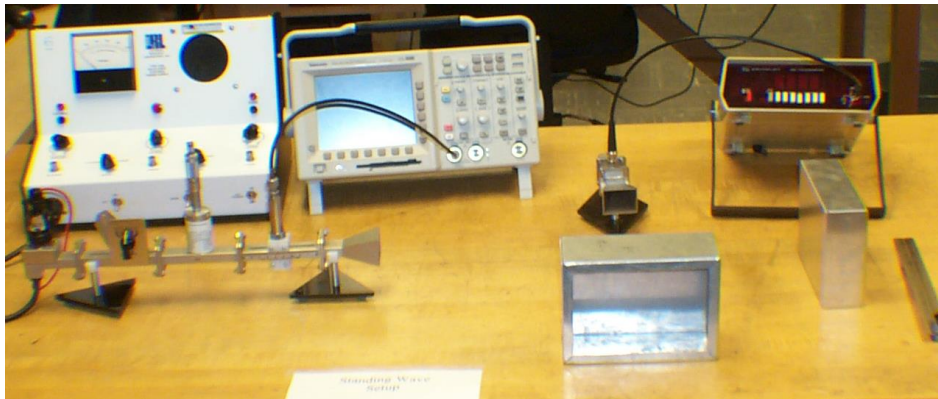
Lloyd's Mirror experiment



Humphry Lloyd
1802-1881

Difference of the wave paths of
“red” and “blue” rays is:

$$\Delta S = \sqrt{h^2 + d1^2} + \sqrt{h^2 + d2^2} - (d1 + d2)$$



Lab setup picture

For constructive interference

$$\Delta S = n\lambda$$



Total internal reflection experiment. Snell's law



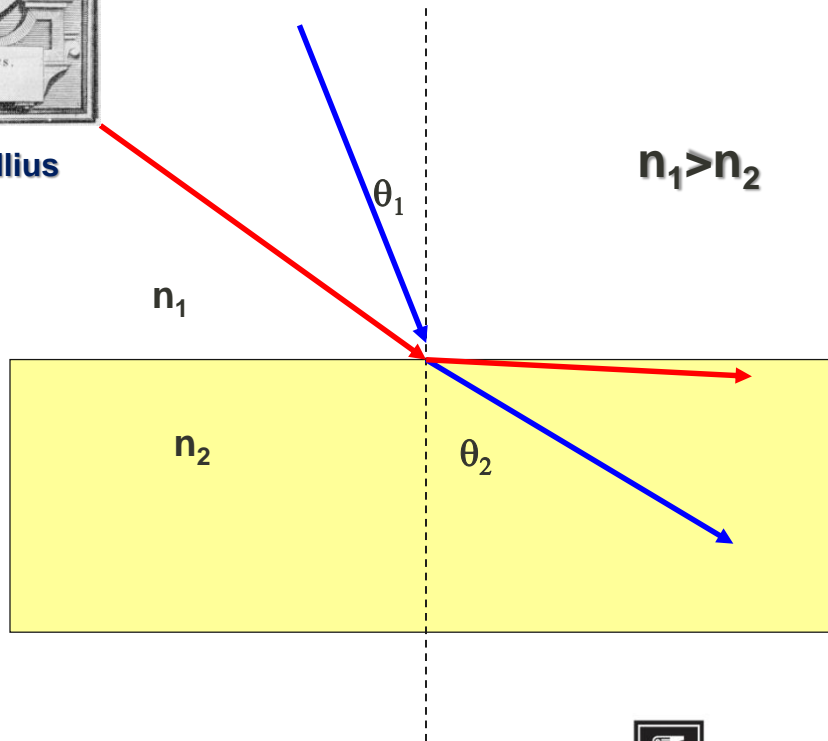
Willebrord Snellius
1580-1626



Claudius Ptolemaeus
after AD 83–c.168)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Snell's law



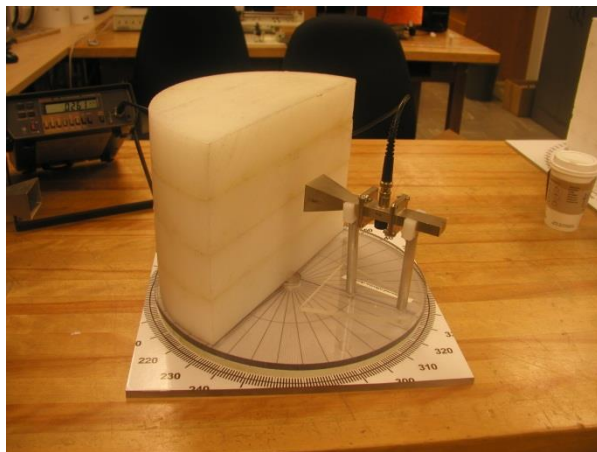
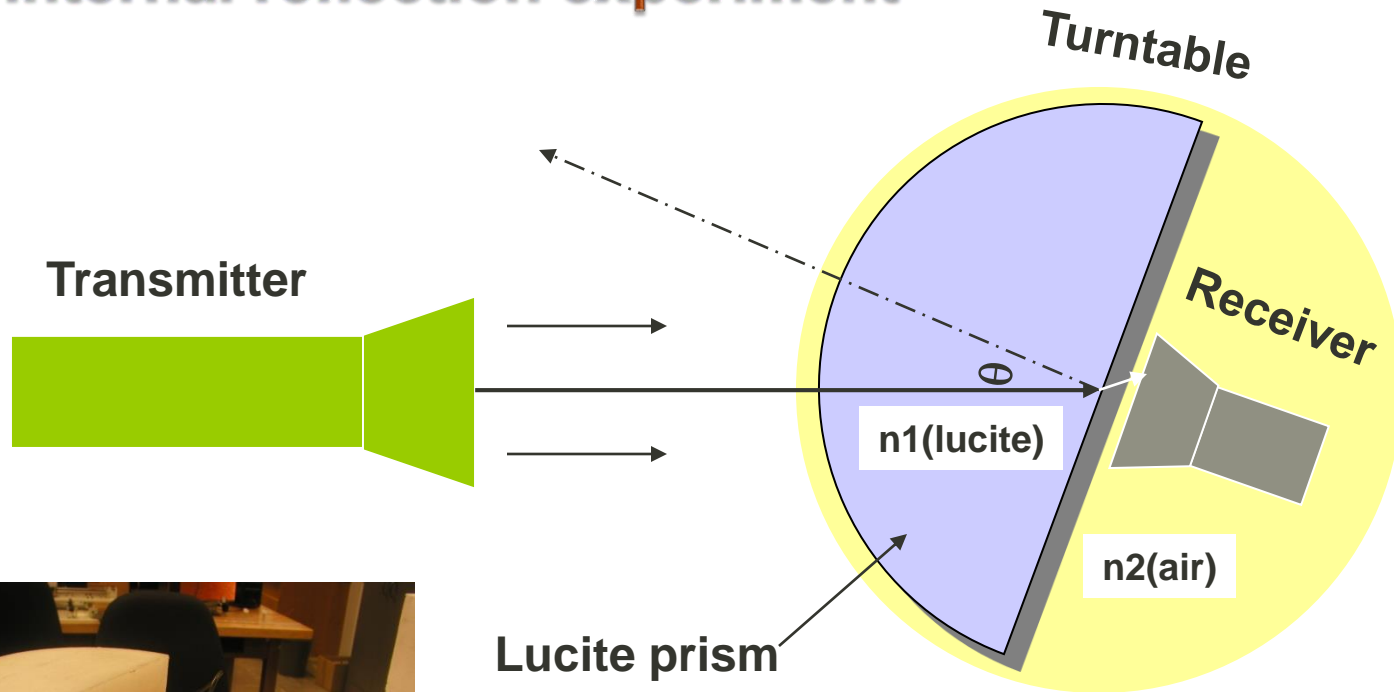
Equation for critical angle:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

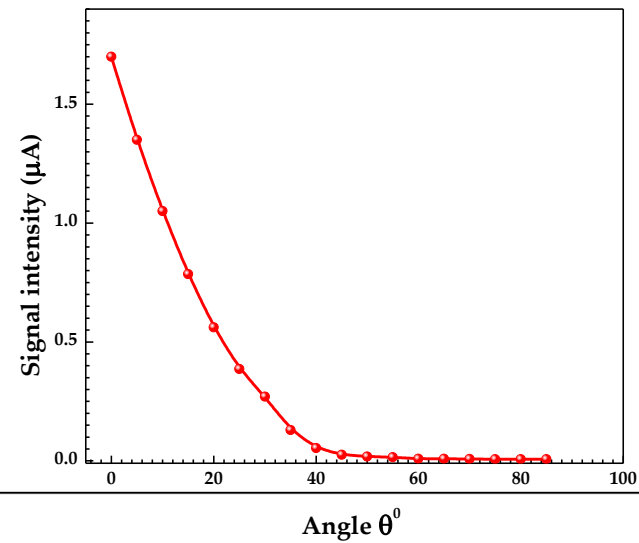
$$\theta_c = \sin^{-1}(n_2/n_1)$$



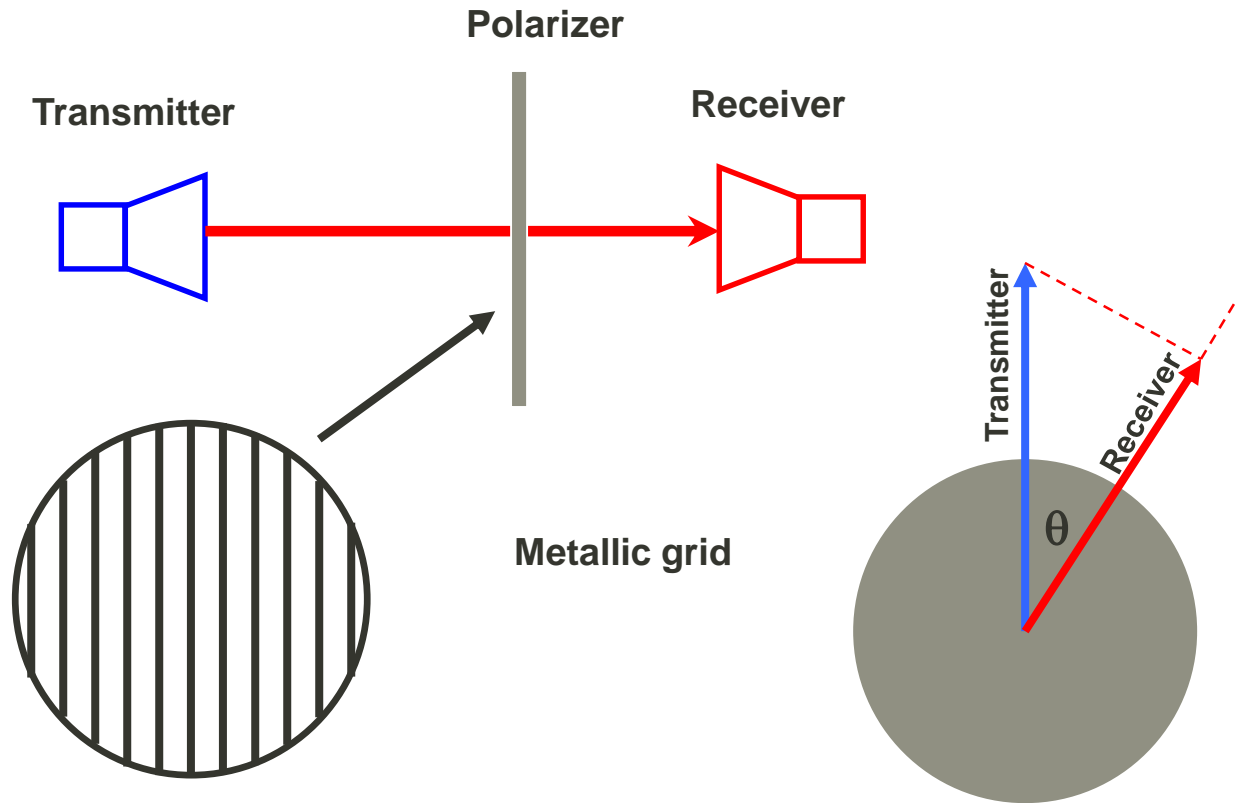
Total internal reflection experiment



Experimental setup and the example of the data



Microwave polarization



Etienne-Louis Malus
1775 – 1812

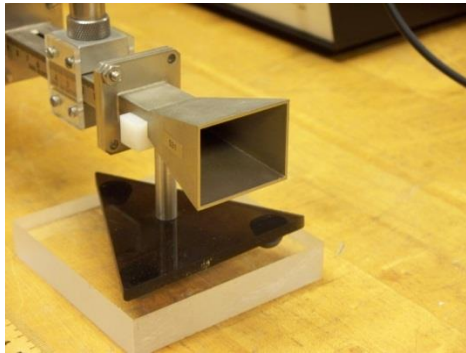
Malus law

$$E = E_0 \cos \theta$$

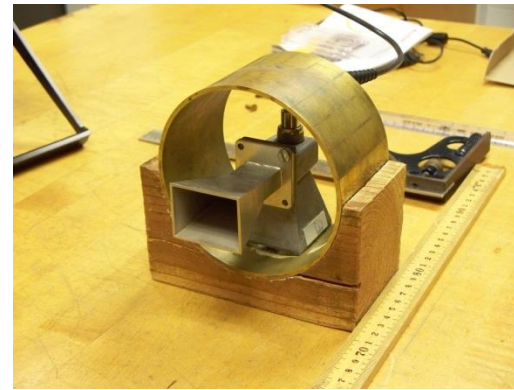
$$I \propto E^2$$

$$I = I_0 \cos^2 \theta$$

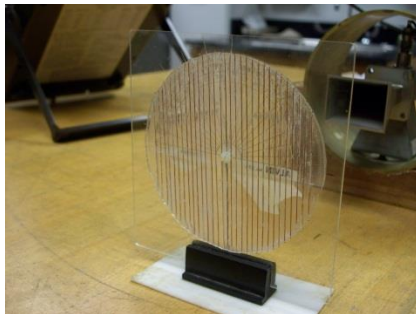
Microwave polarization



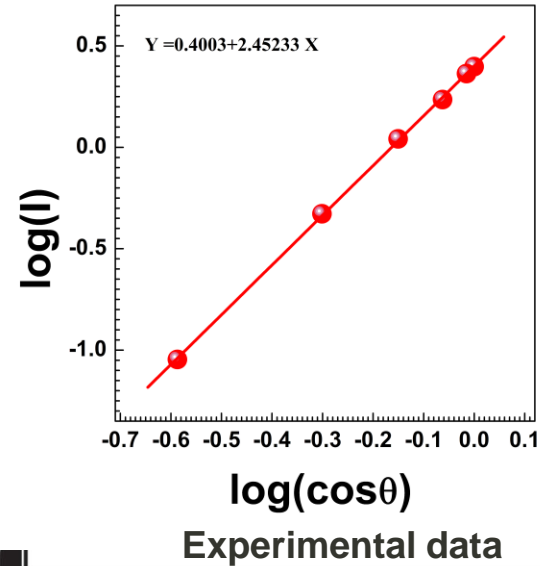
Transmitter



Rotatable receiver



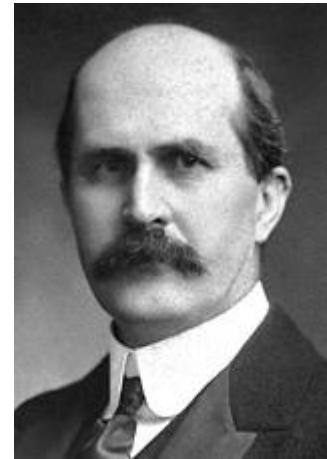
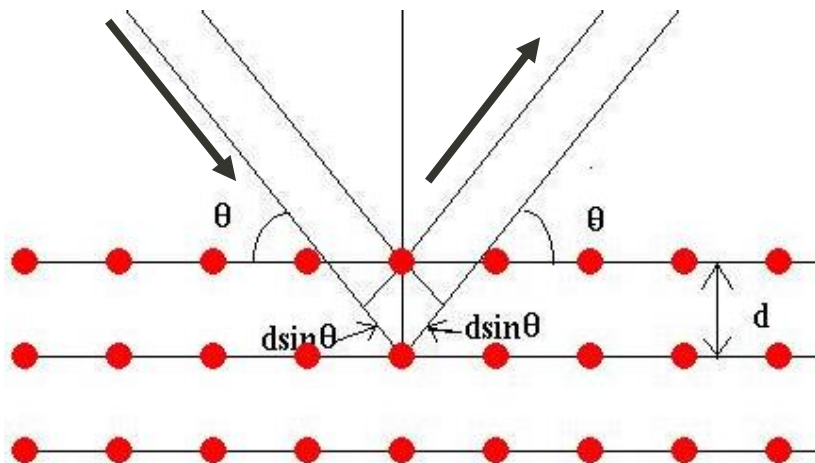
Polarizer



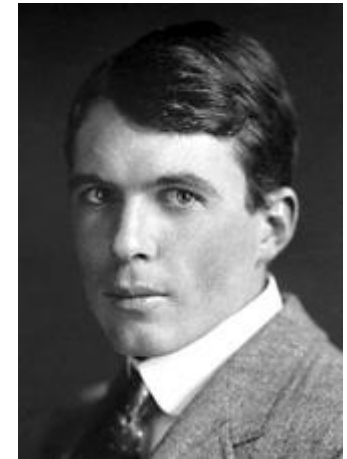
$$I = I_0 \cos^2\theta$$

Bragg diffraction

Interference of the EM waves reflected from the crystalline layers



Sir William Henry Bragg
1862-1942



William Lawrence Bragg
1890-1971

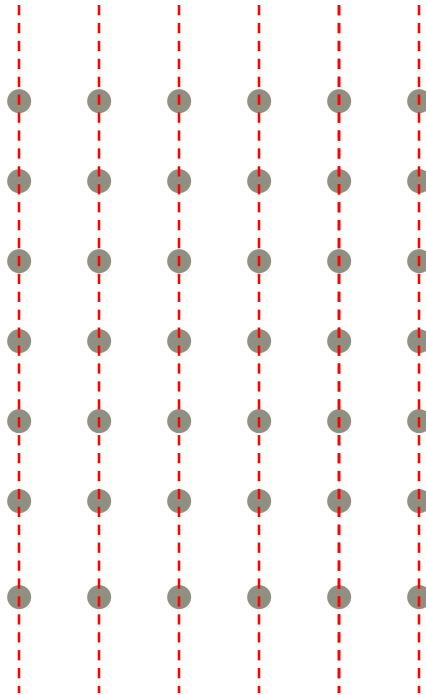


The Nobel Prize in Physics 1915
"for their services in the analysis of
crystal structure by means of X-rays"

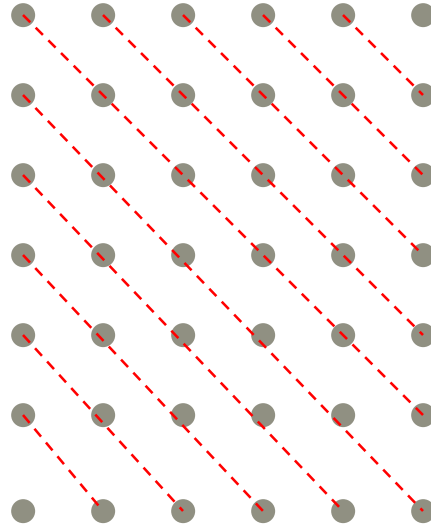
$$n\lambda = 2d \sin \theta \quad \text{Bragg's Law}$$



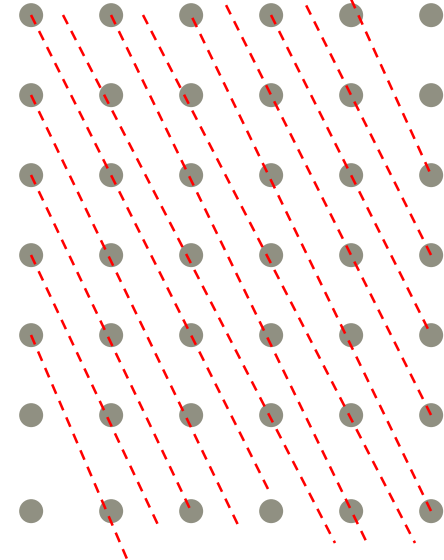
Bragg diffraction



(100)



(110)



(210)

Different orientations of the crystal



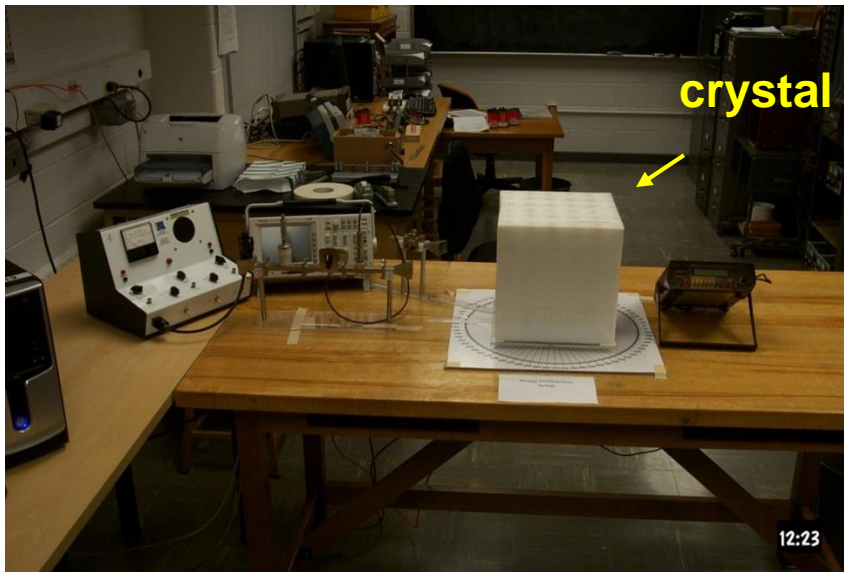
Bragg diffraction

$$n\lambda = 2d \sin \theta$$

$$\lambda < 2d$$

In our experiment $\lambda \sim 3\text{cm}$;
For cubic symmetry the
angles of Bragg peaks
can be calculated from:

$$\left(\frac{\lambda}{2d} \right)^2 = \frac{\sin^2 \theta}{h^2 + k^2 + l^2}$$

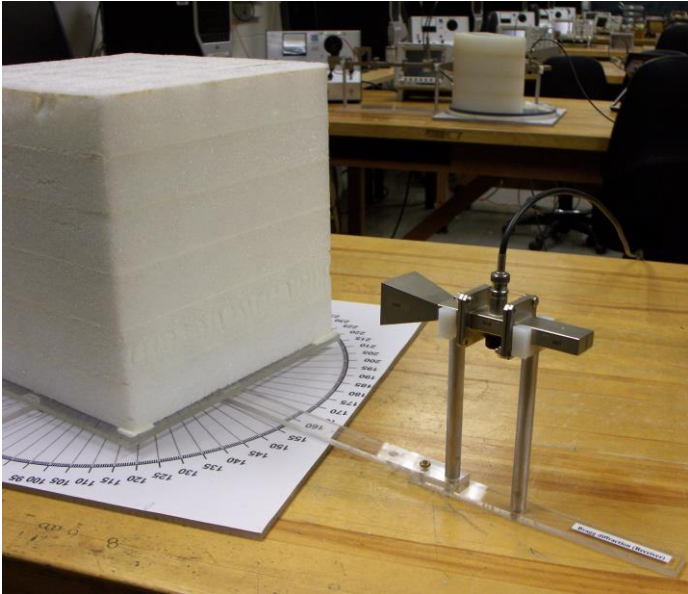
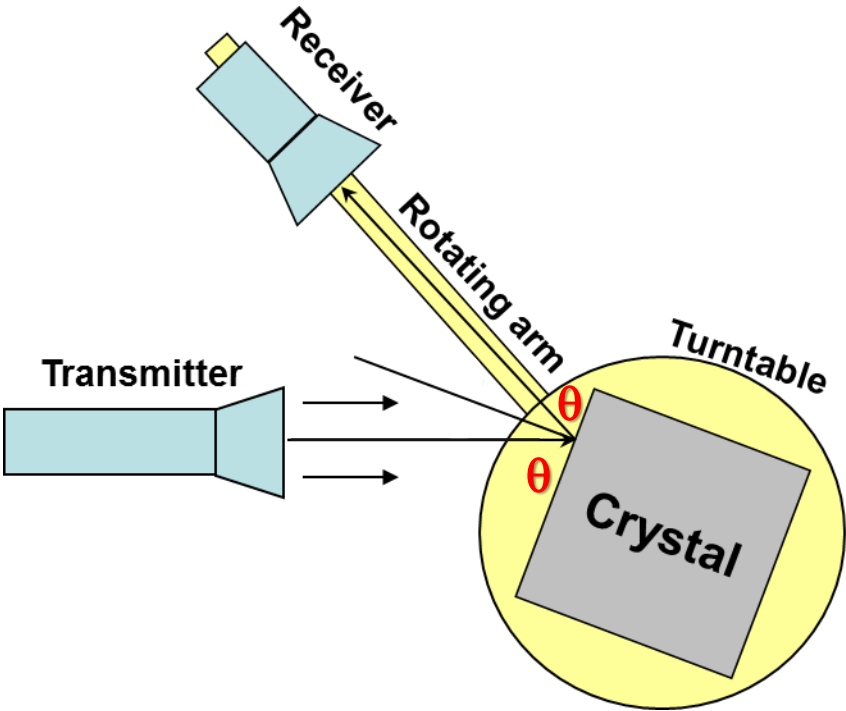


Experimental setup

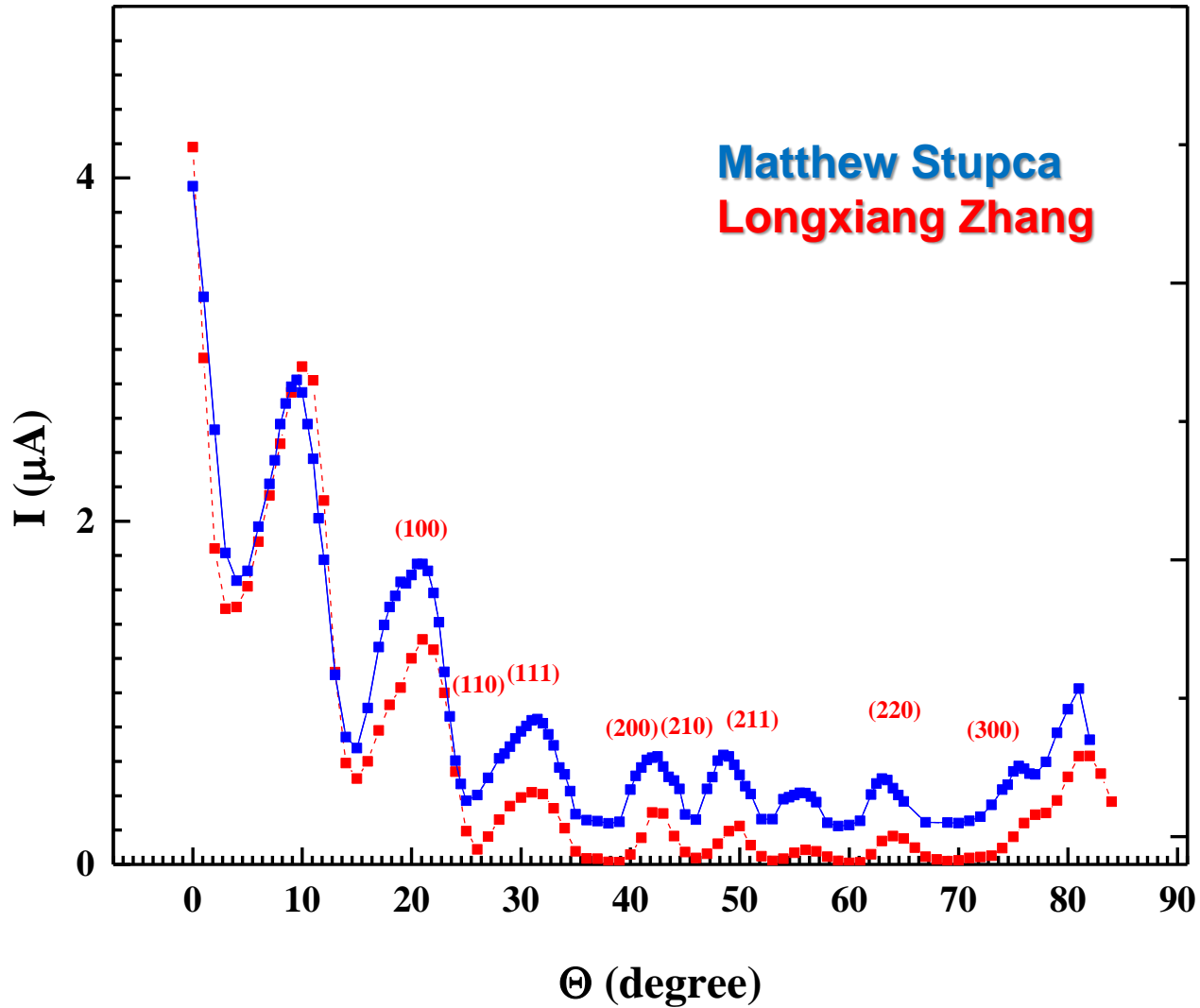
where h, k, l are the Miller Indices.
For crystal with $d=5\text{cm}$ and $\lambda=3\text{cm}$
the 3 first Bragg peaks for (100)
orientation can be found at
angles: $\sim 17.5^\circ$; 36.9° and 64.2°



Bragg diffraction

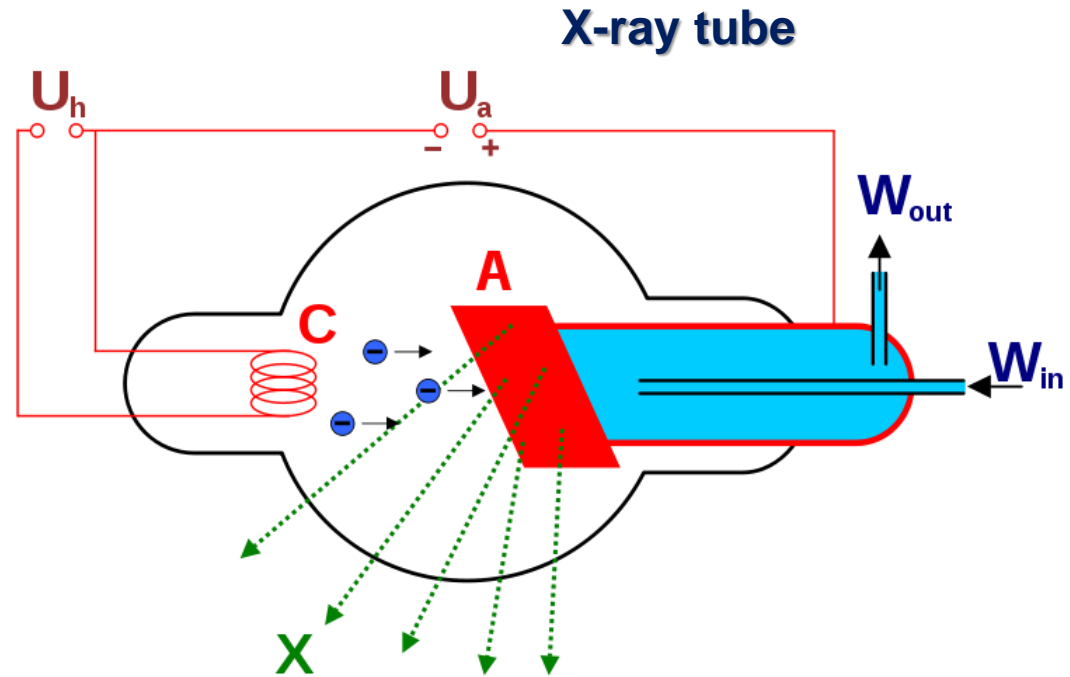
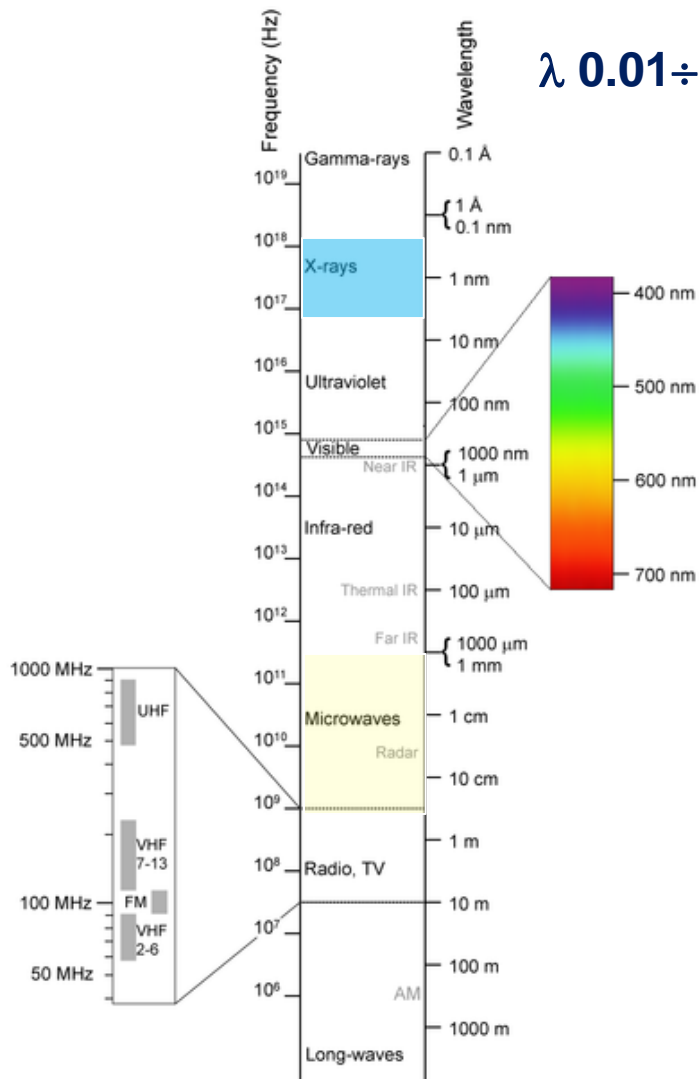


Bragg diffraction. Results.*



Bragg diffraction. X-rays.

λ 0.01 ÷ 10 nm



Bragg diffraction. X-rays.

X-ray K-series spectral line wavelengths (nm) for some common target materials

Target	$K\beta_1$	$K\beta_2$	$K\alpha_1$	$K\alpha_2$
Fe	0.17566	0.17442	0.193604	0.193998
Co	0.162079	0.160891	0.178897	0.179285
Ni	0.15001	0.14886	0.165791	0.166175
Cu	0.139222	0.138109	0.154056	0.154439
Zr	0.70173	0.68993	0.78593	0.79015
Mo	0.63229	0.62099	0.70930	0.71359

David R. Lide, ed. (1994). *CRC Handbook of Chemistry and Physics 75th edition*.
CRC Press. pp. 10–227



*courtesy of Matthew Stupca

Bragg diffraction. X-rays.

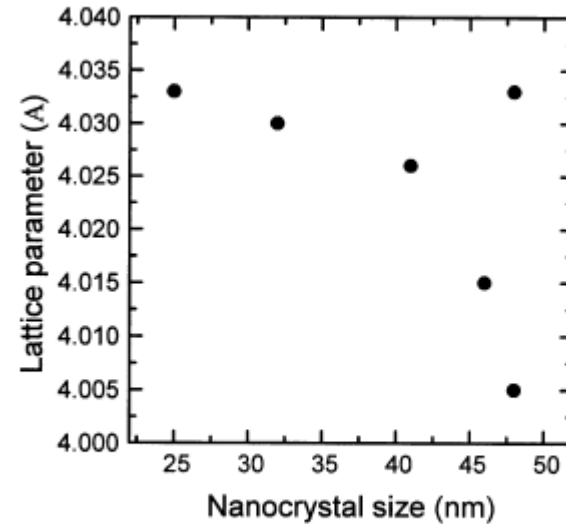
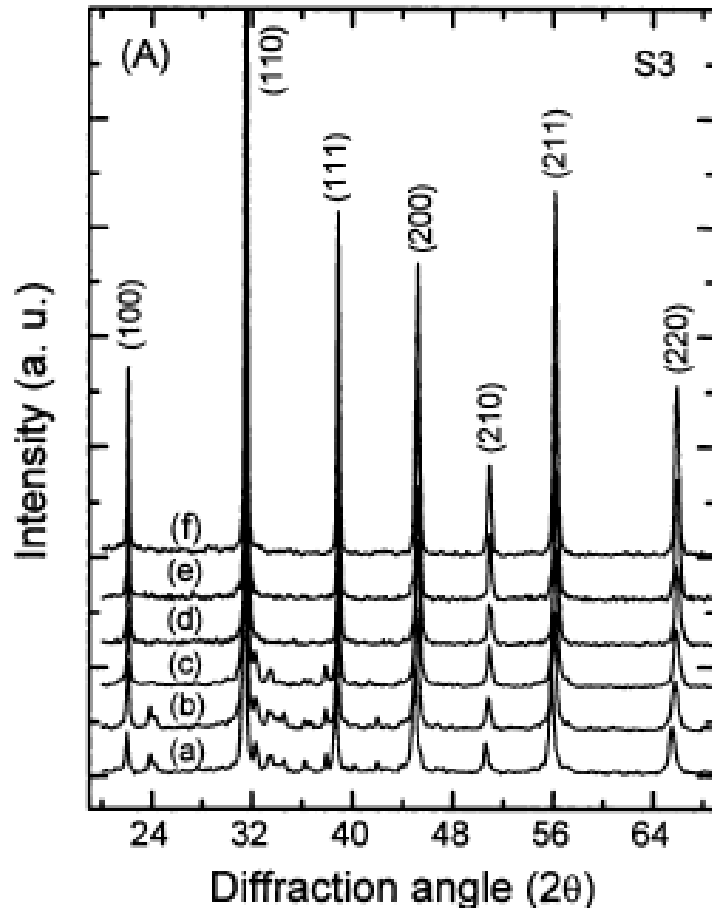


Fig. 4. Lattice parameter c versus the grain size in the BaTiO₃ nanocrystal.

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Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

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Comments and suggestions

1. Klystron is very hot and the high voltage ($\sim 300\text{V}$) is applied to repeller.
2. You have to do 6 (!) experiment in one Lab session – take care about time management. The most time consuming experiment is the “Bragg diffraction”.
3. Do not put on the tables any extra stuff – this will cause extra reflections of microwaves and could result in smearing of the data.
4. This is two weeks experiment but the equipment for the week 2 will be different. Please finish all week 1 measurements until the end of this week

Good luck !

